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Auroral Measurements and Upper Atmospheric Physics

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Many of the most spectacular features of the space environment can be described in terms of basic physical processes that have been studied theoretically or measured extensively in the laboratory. In most large-scale space phenomena, however, the physics required to describe the observations involves many fields of basic research. One of the principal problems of the space physicist is to reduce the geophysical data to familiar concepts. A major difficulty is that in many large-scale phenomena many events occur simultaneously, so that it is not clear if cause-and-effect relations exist among the observed phenomena. An example of a space physics subject in which many basic physics fields are represented and in which cause and effect are not easily distinguished is the polar aurora. A survey experiment will be discussed in which nearly simultaneous measurements were made of the particle flux and spectrum above the atmosphere, of the luminosity, and of the electron densities in the atmosphere. The experiment consisted of measurements carried out in satellites using the techniques of nuclear physics, of optical spectroscopy, and of radio propagation. From this and similar experiments, it is hoped that the role of energetic charged particles in producing auroras can be understood.

Introduction

A SET of coordinated observations on polar auroras using an instrumented satellite and airborne and ground-based observation stations in Alaska has been completed recently. This experiment illustrates the contribution that the aerospace sciences can make to basic physics research. Such research can, in turn, make contributions to rocket and satellite techniques, especially to a better understanding of their communications and control problems and of radiation environments through which astronauts must travel.

Space research near the earth is primarily concerned with determinations of the fluxes of incoming particles and electromagnetic radiations and the interactions of these fluxes with the earth's atmosphere and magnetic field. Some of the particles coming into the atmosphere are protons and electrons from the sun. They travel from the sun as neutral plasmas, large groups of particles made up of equal numbers of positive and negative charges. How they travel from the boundary of the magnetosphere to the atmosphere is not well understood. The known electromagnetic radiations coming into our atmosphere from solar and cosmic sources cover the range of wavelengths from gamma rays, x rays, ultraviolet, visible, infrared, and microwaves to radio waves. The interactions of this wide variety of radiations with the atmosphere provide subjects for a wide field of basic studies.

The uncharged particles and electromagnetic radiations are not affected by the earth's magnetic field, but if they have sufficient energy, they ionize and excite the molecules and atoms of the atmosphere. The low energy radiations heat the atmosphere, or alternately, if the atmosphere is sufficiently transparent to them, the earth. The charged particles are constrained by the earth's magnetic field to approach the earth in zones dependent upon their masses and energies. Neutral plasmas, if they are sufficiently intense, can penetrate into the earth's magnetic field. Through interactions with the earth's field and the residual atmosphere, they may add charged particles to the trapped radiation belts or deposit energy in the atmosphere. The charged particles produced as secondaries by the neutral particles and electromagnetic radiations also contribute to the radiation belts and to energy deposition into the atmosphere.

The determination of particle and electromagnetic radiation fluxes, as well as the interaction of radiations with matter and magnetic fields, have been major concerns of physicists for many years. They have been interested in the use of ionization chambers and scintillation-photomultiplier assemblies as calibrated detectors. In most of the interactions studied, the materials have had relatively high densities, 1 g/cm^3 to 10^{-5} g/cm^3, and the particle ranges have been short, ranging from a few microns to about a meter.

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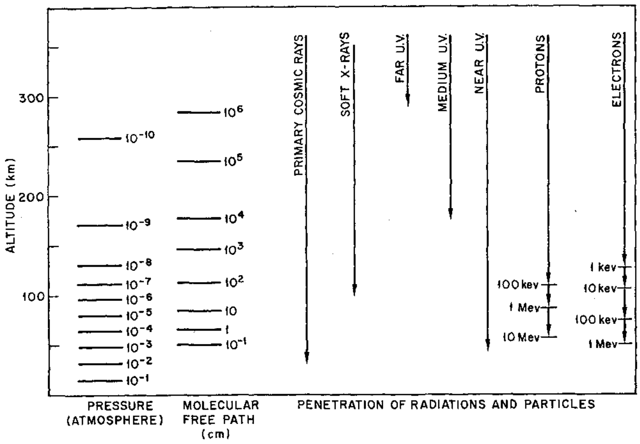

Fig. 1 Approximate depths of penetration of several types of electromagnetic radiations and particles into the atmosphere.

In the field of plasma physics, different types of calibrated detectors have been developed. Among them are microwave probes and interferometers, ion and electron probes such as Langmuir probes, and the instruments of spectroscopy. The mechanisms involved in these detectors have provided subjects of great interest to the basic physics of plasmas. The detectors themselves, therefore, have been studied with great interest, as well as having been developed for use as calibrated detectors. When the interactions of space radiations with the earth's atmosphere and magnetic field are considered, the atmosphere itself is used as a radiation detector, as a tool for studying the basic physical processes. Radiation detection in the atmosphere is accomplished by the ionization and luminosity resulting from space radiation. Space radiations have ranges in sea-level air from 10^{-4} cm (10^{-7} g/cm^2) for far ultraviolet, to 10^4 cm (10 g/cm^2) for primary cosmic rays. The exponential variation of atmospheric density with height allows excellent range and, hence, energy determinations for space radiations. Figure 1 shows the maximum penetration depths of several space radiations into our atmosphere.

Basic Physics Studies in the Atmosphere

The basic physical processes to be discussed are the production mechanisms for excitation and ionization for space radiations and the corresponding decay mechanisms. The production mechanisms are photon, electron, ion, and neutral particle collisions with the constituents of the upper atmosphere. Decay mechanisms for the ions involve charge exchange, electron-ion recombination, ion-ion recombination, and electron attachment. Decay mechanisms for the excited states include natural radiative decay and collisional de-excitation. A striking example of collisional deactivation of the OI state, which decays with the emission of the 6300A line, is apparent in the fact that this emission can occur only at very high altitudes where the time between collisions is long. The lifetime of the excited state by radiation is 100 sec. At lower altitudes (or in the laboratory), collisions with other gas molecules, or with chamber walls, deactivate this level before radiation can take place.

A limited investigation of these mechanisms can be carried out in the laboratory with sea-level composition air, but only down to densities such that the mean free paths of the air molecules remain shorter than the dimensions of the container. However, above altitudes of 100 km, the composition of the atmosphere changes from molecular to atomic, and mean free paths extend from a few meters to many kilometers as the altitude increases.

Measurements of ion and electron concentrations and production rates are necessary in order to determine ion recombination rates. Electron concentrations can be measured by attenuation and phase shift measurements of radio frequency electromagnetic radiation or by electron collection techniques. Both of these methods can be used in the laboratory for sufficiently high electron concentrations. Experimental regions for such electromagnetic radiation propagation studies must be of the order of one wavelength of the radiation being used. This limits laboratory studies to essentially microwave frequencies, i.e., radiations with wavelengths less than a few meters. Measurable effects at such frequencies require electron densities greater than 10^9 electrons/cm^3 (for which the plasma frequency = 285 Mc and λ = 1 m). However, electron densities down to a few electrons per cubic centimeter (for 10 electrons/cm^3 the plasma frequency is 28.5 kc and λ = 10.5 km) can be measured in regions that have dimensions of 10 km or more. Thus, the large dimensions of the space laboratory give us room to study recombination rates in ranges that would be impossible to study effectively in our sea-level laboratories.

The polar aurora illustrates a phenomenon that is suitable for studying the problem of the interaction of space radiation with the earth's magnetic field and atmosphere. Simply described, auroras are regions of luminosity and ionization produced by electrons and protons following magnetic field lines into the atmosphere. There is evidence that there is a wide variation of particle energies between different auroras which produces an extensive variety of visible phenomena. In many cases, auroras are sufficiently well localized to facilitate height measurements. In order to understand the basic procedures that were used to carry out the recent auroral investigation with an instrumented satellite, it will be necessary to describe some of the main characteristics of auroral phenomena.

Characteristics of Auroras

Auroras are luminous regions in the upper atmosphere resulting from a high level of excitation and ionization of its atoms and molecules by high energy electrons and protons. The luminosity is such that auroras can be observed visually only at night. They occur almost every night in the auroral zones, which are bands a few hundred miles wide, along lines of constant geomagnetic latitudes about 67° north and south. This band where auroras occur with maximum frequency in the northern polar region passes near Fairbanks, Alaska; Fort Churchill, Canada; the southern tip of Greenland, Ice-

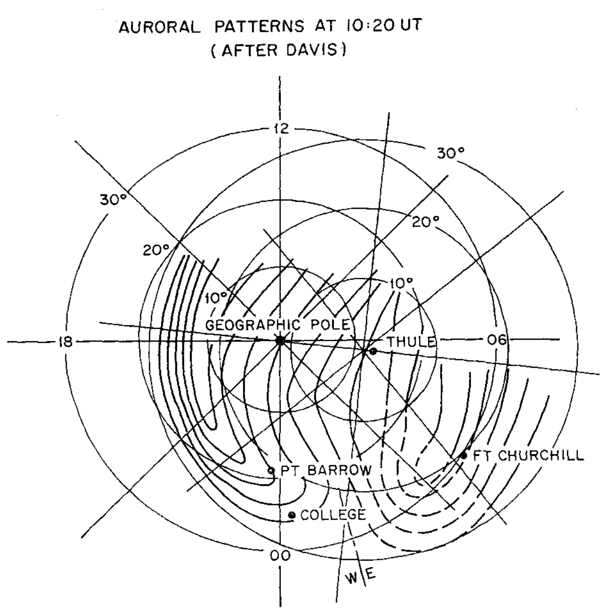

Fig. 2 Auroral patterns at 10:20 UT (after Davis[4]). The circles with centers near Thule, Greenland are magnetic colatitude lines.

land; and Tromsö, Norway. Auroras are observed to occur at lower latitudes during large solar storms, but the frequency is less away from the auroral zones.

Although auroras occur in a variety of forms and intensities, they fit into a few more or less well-defined types. Detailed discussion of these types and photographs of typical auroras in the auroral zones can be found in three recently published books.[1-3]

The quiet homogeneous arcs are the most suitable subjects for the present measurements. These arcs are luminous sheets aligned along lines of geomagnetic latitude extending, in most cases, down to altitudes of 80 to 125 km. In width, they may extend for hundreds of kilometers along the latitude lines; they may be quite thin (1 to 10 km) in a direction perpendicular to latitude lines. The heights of the main luminous bodies of the sheet (to half-peak luminosity) extend upward from 10 to 30 km above their base.

Auroras have typical life cycles. Before midnight local time, measurements indicate a general pattern of parallel luminous sheets, which are well separated. They may be homogeneous arcs, arcs with rayed structures, or a diffuse glow with about the same dimensions as those just given.[4] After midnight, some vestiges of the before-midnight pattern remain, but the luminous areas, now somewhat smaller, pulsate and move about more rapidly. The small broken-up arcs have higher luminosities, incident fluxes, and electron densities than the quiet arcs. Their characteristics make them less well adapted to quantitative observations. The use of rapid scan-photometers, however, makes many isolated and reasonably slow-moving luminous sheets suitable experimental subjects. The patterns of the auroral forms in the vicinity of the geographic and geomagnetic polar areas can be similar to those shown in Fig. 2. The solid lines represent fairly stable arc-type formations, and the dotted lines show the general areas and orientations of the auroras of the rapidly fluctuating type. For the experimental measurements to be described here, the satellite passed over the Alaskan region near the breakup stage.

Survey Experiment to Study Auroras

The direct data sought in this experiment were 1) the incident fluxes, energy spectra, and angular distributions of the particles (mainly electrons and protons) coming into the region of the aurora; 2) the vertical distribution of auroral luminosity with absolute luminous fluxes of selected wavelengths, which could be interpreted in terms of production rates for certain ions such as N_2^+, for which the production cross section is known, and for prominent excited states in N_2^+, N_2, O, and H; 3) measurements of the time and space uniformity of the luminosity over the ranges pertinent to the coordinated particle and ionization measurements; and 4) radio attenuation and relative phase measurements on a beam from a multifrequency beacon to determine the electron concentrations and the location and shape of the ionized regions in the vicinity of auroras.

The main ideas involved in the measurements are shown in Fig. 3, a schematic diagram showing a section through an aurora and along a geomagnetic meridian plane. The plane of the auroral curtain is almost perpendicular to the satellite path. A satellite carried the package of particle detectors through the region above the luminous sheets where the particle fluxes and energies were measured. The data were telemetered to the Kodiak ground station. All the instruments were to be read about 100 times a second to determine accurately the shape of auroras, since the satellite passes over them in a few tenths of a second. The scanning optical telescope was included to measure the amount of light and its vertical distribution. As the satellite moved, the radio beam swept through the ionized regions associated with the aurora and suffered attenuation.

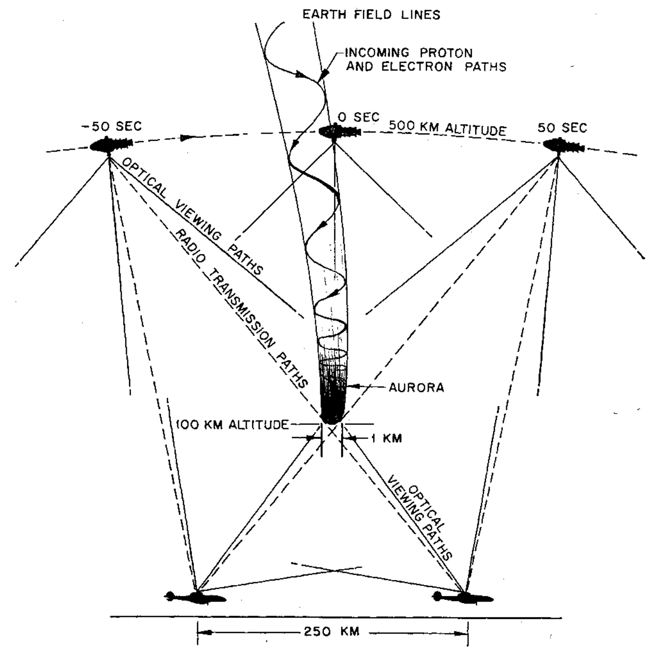

Fig. 3 Schematic diagram of survey measurements on auroras.

The airborne observatories were necessary to get above the cloud cover and to be underneath the satellite paths for good geometry measurements. The B-17 aircraft was supplied by Aero Service Corporation of Philadelphia, Pennsylvania and was based at Fort Wainwright (formerly Ladd Air Force Base) near Fairbanks, Alaska. The B-17 carried 20- and 40-Mc radio receiving gear and recorders, as well as a five-channel optical photometer and automatic cameras.

The optical equipment, five-frequency optical scanner and fore-and-aft pointing automatic cameras, was mounted on a stabilized platform and looked through what was formerly the hatch in the top of the radio operator's compartment. An experienced auroral investigator from the University of Alaska, Wallace Murcray, used the astrodome in the navigator's compartment for visual observations and described over the intercom system and onto a tape recorder the auroral activity through the data-taking runs.

The KC-135 experimental aircraft of the Geophysical Research Directorate of Air Force Cambridge was used to assist in the optical measurements. This aircraft was equipped with a scanning photometer, automatic camera, spectrograph, and all-sky camera.

The Geophysical Institute of the University of Alaska operated 20- and 40-Mc radio receiving equipment for attenuation, phase comparison, and interferometer measurements at College. They operated their scanning optical photometers, automatic cameras, all-sky cameras, and spectrographs at their College and Fort Yukon ground stations. They also operated all-sky cameras as Bettles, Kotzebue, and Point Barrow, Alaska.

Figure 4 shows a map of Alaska, exhibiting the spatial scope of the experiment. The locations of the ground stations and satellite orbits for which data were obtained are marked. The six nearly vertical arrows near the auroral zone show the location of the six data-taking runs of the B-17 aircraft. Auroras were observed on all flights, and the aircraft was above cloud cover all six nights. Most missions were flown at 10–12,000 ft, but one mission was flown at 18,000 ft. The four nearly horizontal arrows show the missions flown by the KC-135 aircraft. The flight path was across the satellite track since observations could be made from the side windows only.

Ground station and aircraft radio receivers made attenuation and phase measurements on the 20- and 40-Mc beams from the satellite-borne transmitter. Interesting particle

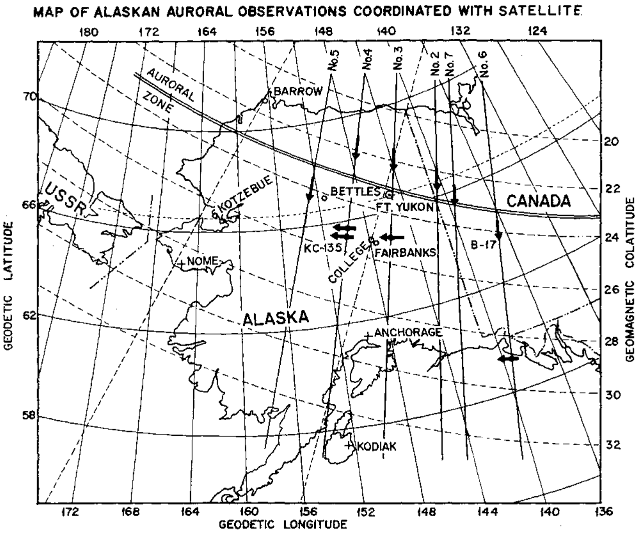

Fig. 4 Map of Alaska showing locations of ground stations and B-17 and KC-135 aircraft missions.

data were obtained from two particle detectors with thresholds of 2 kev and 28 kev for electrons. Data from the other instruments and from the fast electronic commutator were lost when a telemetering link failed before the experimental turn-ons occurred.

A list of the instruments flown on the satellite and their characteristics is given in Table 1. A photograph of the satellite package is shown in Fig. 5. Most of the particle detectors look upward at angles within 60° from vertical (which is to the right on the photograph). The topside scanning optical telescope and the radio beacon with its 10-ft carpenter's tape antenna were mounted on the lower side of the panel.

Results

The best set of coordinated measurements were obtained on flight 3 (Fig. 4). There was a long relatively stable isolated homogeneous arc just south of Fort Yukon. Both particle detectors gave readings above their background levels for about 18 km.[5] Curves a and b of Fig. 6 show these readings for the 2- and 28-kev threshold detectors, respectively, across the arc. Under the assumption that the electron spectrum had an exponential shape of the form $N(E)dE = A \exp(-E/E_0)dE$, the electron flux at the peak was found to be 10^8 electrons/cm²-sec-sr, and the characteristic or average energy E_0 of the spectrum varied from 6 to 9 kev at different points across the arc (Fig. 6, curve c). Belon, Romick, and Deehr, of the University of Alaska, measured the luminosity profiles of the arc with ground-based scanning photometers at College and Fort Yukon. Their data are presented in Ref. 6. The total energy

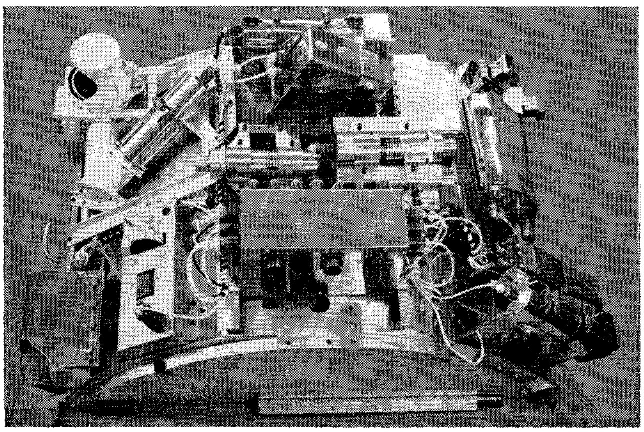

Fig. 5 Satellite instrument package.

Table 1 Particle detectors in satellite package

Instrument		Direction[a]	Energy range	Energy intervals
Proton spectrometer	I	13° ± 20°	50–4000 kev	5
	II	48° ± 20°	50–4000 kev	5
Electron spectrometer (Scintillation	I	13° ± 20°	50–200 kev	5
crystal spectrometer)	II	50° ± 10°	50–200 kev	5
Electron spectrometer	I	13° ± 5°	20–160 kev	4
(GM tube spectrometer)	II	60° ± 5°	20–160 kev	4
	III	180° ± 3°	20–160 kev	4
Total energy detectors	I	13° ± 30°	2 kev	1
	II	13° ± 30°	10 kev	1
	III	13° ± 30°	28 kev	1

[a] The directions in which the instruments are pointing are measured relative to the earth's magnetic field lines. Instruments with a direction of 0° will accept particles traveling down along field lines toward the surface of the earth. The listed angular ranges are for the extreme acceptance angles (full width of the base of the resolution function).

deposit needed to account for their luminosity profile was about 200 ergs/cm²-sec.

The high value was surprising since the electron data in Fig. 6 indicated that less than 5 ergs/cm²-sec was precipitated by electrons with energies greater than 2 kev. These two different observations lead to the conclusion that for this particular case, which was recognized as an unusual aurora with a peak luminosity at 145 km, all but a few percent of the energy was deposited by electrons with energies below 2 kev. The differential Doppler measurements on 20 and 40 Mc by Owren and Hook[6] gave total integrated electron density measurements ($\sim 2 \times 10^{12}$ electrons/cm² column from 200 to 275 km altitude) along the beam path from the receiver to the transmitter. The radio propagation and luminosity data can be combined to give a volume recombination coefficient by making reasonable assumptions about the stability of the

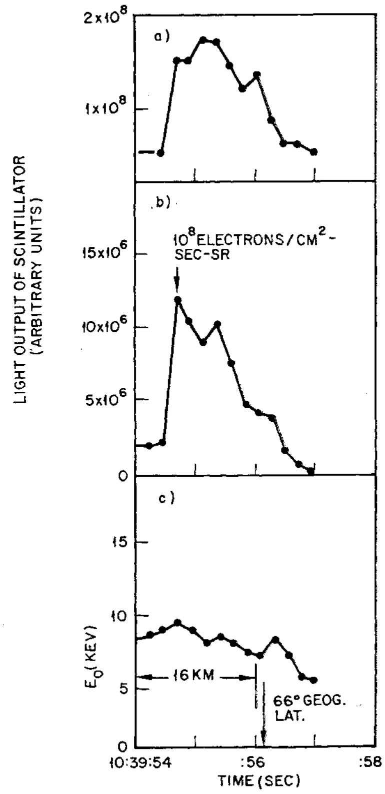

Fig. 6 Scintillator light output on pass 3 for two total energy detectors; a) 2-kev threshold detector, b) 28-kev threshold detector, and c) plot of the average energy E_0 of the exponential spectrum derived from each pair of 2- and 28-kev threshold detector data.

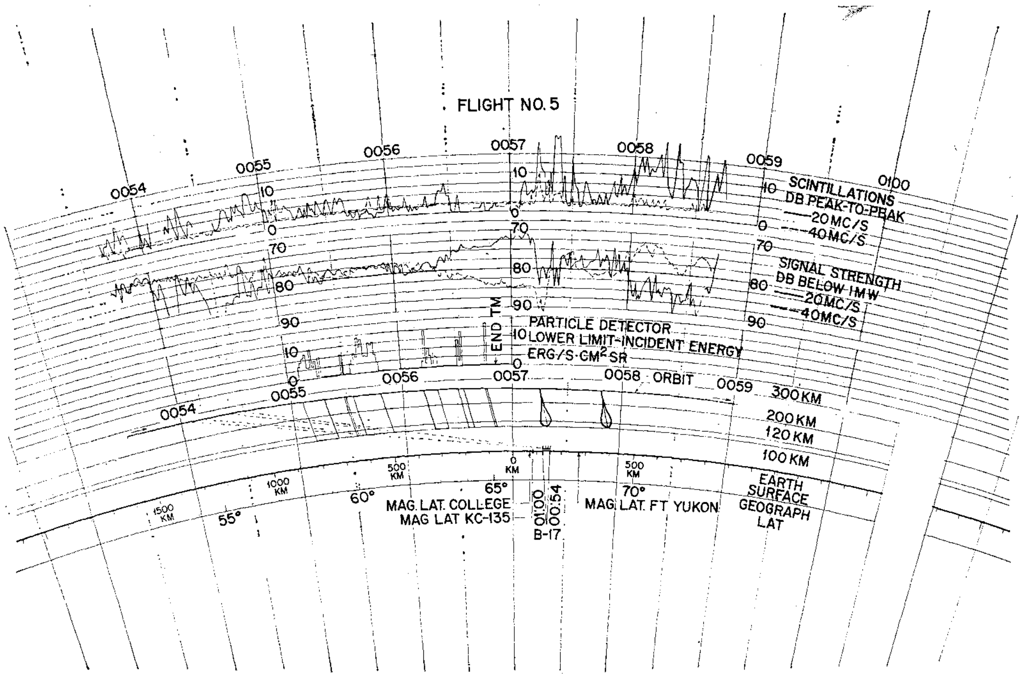

Fig. 7 Satellite particle data, B-17 optical data, and B-17 radio data plotted as a function of satellite location in the orbital plane for pass 5, 10:57 UT, March 3, 1962.

aurora between luminosity scans and about the distributions of luminosity and ionization outside the visible auroral form. The value of 10^{-6} cm^3/sec for the recombination coefficient in the 200- to 275-km altitude region is obtained from these data. This result must be considered an order-of-magnitude value, but for the recombination coefficient at these altitudes, order-of-magnitude values are worthwhile. Another finding from a combination of the particle, luminosity, and electron density measurements was that appreciable amounts of luminosity and ionization could occur outside the sharp auroral forms and that if these effects were caused by electrons, those electrons had to be very "soft" for the case measured.

Another interesting case was flight 5 during which particle data from the satellite and radio signal strength data from receivers in the B-17 aircraft were coordinated. There were no optical data for the regions where particles were observed. The two cameras on the aircraft were pointed at the more spectacular forms overhead and to the north as shown by the teardrop-shaped representations in Fig. 7. As the satellite moved north along the orbit, the 2-kev threshold particle detector encountered elevated particle fluxes in well-separated regions. The energy spectra were soft and steep since only the 2-kev threshold detector gave readings. If the spectrum is assumed to have an exponential shape and to be just below the calibration level of the 28-kev detector, a lower limit on the deposited electron energy is obtained. These values are plotted in Fig. 7 just above the orbit, and the trapezoids represent the region of space where the magnetically confined charged particles would interact with the atmosphere to produce luminosity and ionization.

The signal strength and low-frequency (<100 cps) scintillation data for 20 and 40 Mc obtained by Moe and Tyer[7] are shown at the top of Fig. 7. There are two places where dropouts of the 20-Mc signal to the B-17 receivers occur, but the 40-Mc signal was not affected. Thus, somewhere along the 20-Mc beam path, the critical ionization density of 5×10^6 electrons/cm^3 for 20 Mc was reached. It will be noted that straight-line beam paths from the satellite to the B-17 aircraft corresponding to the dropout regions pass through two of the large regions of soft-particle precipitation below 150 km. The energy precipitated by the electrons above 2 kev should produce 5×10^{11} ion pairs/cm^2-sec; most of these should be deposited in a 20-km layer. These values combine to give a lower limit to an E-layer recombination coefficient near 10^{-8} cm^3/sec. Large increases, consistent with observations, could result if there were many electrons below 2 kev or if the dropouts were caused by high-intensity filaments.

Radio-signal-strength data obtained by the B-17 borne receivers on this flight, and also on flight 3 discussed earlier, showed inhomogeneities in the north-south direction in the ionosphere below the satellite height with characteristic sizes about 250 m. Although these inhomogeneities cannot be related directly to inhomogeneities in particle precipitation in this set of measurements, it is interesting to note that order-of-magnitude fluctuation in particle intensities did occur in 0.2-sec intervals in flight 5. The detailed particle data[5] shown in Fig. 8 is for the active auroral region near 10:55:40 universal time on flight 5 (Fig. 6) in which one of the 20 Mc dropouts occurred. Adjacent data points are 0.2 sec apart and are also separated by 1.6 km of satellite travel. It is not possible to separate dependence on time and distance in the present measurements.

The measurements described here have demonstrated the value of the coordinated measurement approach on space-related, large-scale geophysical measurements. The instrumented aircraft can play a versatile role in performing good geometry measurements with satellites over water, above

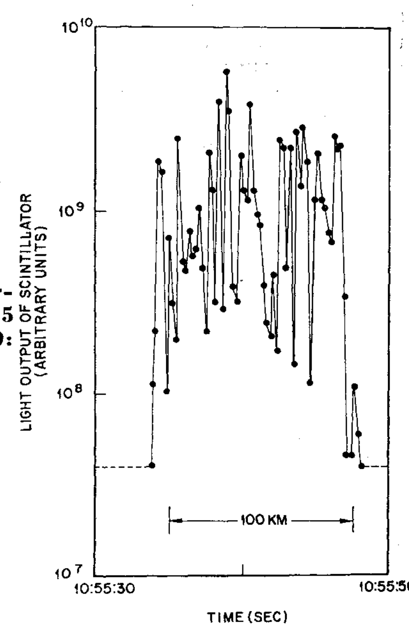

Fig. 8 Detailed particle data for pass 5 from 10:55:30 to 10:55:50 UT.

cloud cover, and in regions where the economics of the situation do not justify the establishment of permanent laboratory facilities.

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Drag Displacements and Decay of Near-Circular Satellite Orbits

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A drag analysis is carried out for near-circular satellite orbits, as used in relay and reentry missions. This work is formulated in terms of moving coordinates that have been previously applied to perturbation studies involving lunisolar gravitation and radiation pressure. The equations of motion for the satellite are written in terms of displacement components relative to an unperturbed Keplerian or "nominal" orbit. These components form an orthogonal triad, whose origin always lies at the nominal satellite position on the elliptic path. The rationale of working from some reference orbit is common to all established techniques of celestial mechanics (Encke, Hansen, Vinti, von Zeipel, et al.). Although our formulation uses the most primitive form of reference trajectory, the calculations remain simple enough to permit an allowance for moderate eccentricity of the nominal orbit and variability of atmospheric conditions. Predictions of orbit altitude and inclination during "spiral decay" are also possible. Perturbations in satellite coordinates, as given here, have obvious applications in guidance work.

1. Introduction

In the analysis of orbital motion of some communication satellites and re-entry vehicles, a typical orbit possesses rather small eccentricities and such altitudes that the predominant perturbations are due to launch errors, the equatorial bulge, and atmospheric drag. The present paper applies a perturbation method that has previously yielded useful results in the study of extraterrestrial gravitation, radiation pressure, oblateness effects, and perturbations of the initial conditions.[1] Some applications of these results have also been made in the study of re-entry trajectories. The perturbative motion of the vehicle is described in terms of displacements from the instantaneous position that it would occupy on a nominal elliptic orbit in the absence of all disturbances. The practical advantage of this formulation for position forecasts and calculations of orbit decay is obvious. The need for such a prediction scheme with low eccentricity has been previously stated in the literature.[2]

If applied to a study of drag perturbations of nominally circular orbits, the computations based on the present method turn out to be surprisingly simple by comparison with the more challenging analyses based on other methods. Consequently, some allowance for the time dependence of atmospheric conditions can be made without appreciable difficulty. If, however, terms of $0(e)$ and $0(e^2)$ are included to account for some eccentricity of the nominal orbit, the algebra becomes more laborious. When rather large eccentricities are to be considered, an approach through Lagrange's planetary equations or by a suitable numerical scheme[3] may be more expedient.

2. Coordinate Systems and General Expressions for the Perturbations

Figure 1 illustrates the moving coordinate system used in the formulation of the present method. The perturbative displacement of the satellite from its nominal position $0'$ is described by the triad of components ξ, η, ζ. The angular argument by which $0'$ is located from the node on the nominal orbit shall be denoted by θ. We have $\theta = \omega + f$ where, for circular nominal orbits, ω marks a convenient reference point from which the independent variable f is measured and the latter increases linearly with time. For elliptic nominal orbits, ω, of course, denotes the nominal argument of perigee and f denotes the true anomaly. The initial position of $0'$ is given by f_0 at $t = t_0$. Once ξ, η, ζ have been found, the transformations from the orbital coordinates to the x, y, z system, and hence to the azimuth-elevation system for tracking radars, are quite straightforward and produce the pointing angles desired as the ultimate goal of this analysis.

Received April 18, 1963; revision received April 16, 1964. The material in this paper was presented orally at the 1960 International Astronautical Federation Congress in Stockholm. Other publications in this area[12–15] show that interest in this facet of satellite motion has persisted, and a written account of the author's effort at this time may not be reduced entirely to the role of a "historical note."

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